Determine if $y = Ax + Be^{-2x} + \frac{x^2}{2}$ is a family of solutions of the DE $(2x+1)y'' + 4xy' - 4y = 4x^2 + 4x + 4$. **SCORE: _____ / 6 PTS**

<u>**State your conclusion clearly.**</u>

$y' = A + x - 2Be^{-2x}$ 1

$y'' = 1 \quad + 4Be^{-2x}$ 1

$$\begin{aligned}(2x+1)y'' \\ +4xy' \\ -4y\end{aligned} = \boxed{\begin{aligned}&1 + 2x + (\cancel{8B}x + 4B)e^{-2x} \\ &+ \cancel{4A}x - (8Bx \quad)e^{-2x} + 4x^2 \\ &- 4Ax - (\quad \cancel{4B})e^{-2x} - 2x^2\end{aligned}}$$ 1

$= 1 + 2x + 2x^2 \neq 4x^2 + 4x + 4$

2

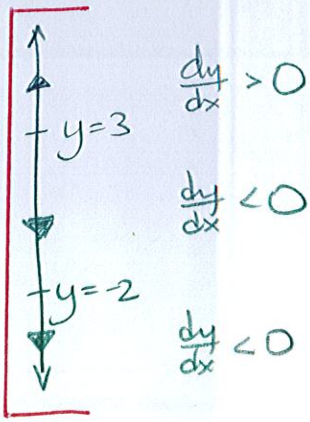
$\boxed{\text{NO}}$ 1

Consider the DE $\frac{dy}{dx} = (y^2 - y - 6)(y+2)$. $= (y+2)^2(y-3)$ **SCORE: _____ / 6 PTS**

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
<u>You must draw a phase portrait to get full credit.</u>

EQ SOL'N $y = -2, y = 3$

1



$y=3$ IS UNSTABLE
1 1

$y=-2$ IS SEMI-STABLE
1 1

[b] If $y = m(x)$ is a solution of the DE such that $m(5) = 1$, what is $\lim_{x \to \infty} m(x)$?

-2 1

Consider the IVP $y' = 2xy^2 - 3x, \quad y(-1) = 2$. Use Euler's method with $h = 0.2$ to estimate $y(-0.6)$. **SCORE: _____ / 4 PTS**

$$y(-0.8) \approx y(-1) + y'(-1)(-0.8 - {}^{-}1)$$
$$= 2 + (2(-1)(2)^2 - 3(-1))(0.2)$$
$$= 2 + (-5)(0.2) = 1 \quad (1) \quad (1)$$
$$y(-0.6) \approx y(-0.8) + y'(-0.8)(-0.6 - {}^{-}0.8)$$
$$\approx 1 + (2(-0.8)(1)^2 - 3(-0.8))(0.2)$$
$$= 1 + (0.8)(0.2) = 1.16 \quad (1) \quad (1)$$

In a certain society, the rate at which a person's wealth changes is proportional to the difference between their wealth and a fixed baseline (call it B, where $B > 0$). If everyone is getting poorer (except for those whose wealth equals the baseline), write a DE for the wealth of a person whose current wealth is half of the baseline. **SCORE: _____ / 4 PTS**

Justify the signs of all symbolic constants (other than B) in your DE properly, but briefly, as shown in lecture.
Do NOT use the absolute value function in your answer.

$W(t)$ = WEALTH @ TIME t

$\frac{dW}{dt} = k(W - B)$ (2)

$W < B$ ("HALF OF BASELINE")

SO $W - B < 0$ $(\frac{1}{2})$

AND $\frac{dW}{dt} < 0$ $(\frac{1}{2})$ ("GETTING POORER")

SO $k > 0$ (1)

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP $(y')^3 - 1 = x + y, \quad y(1) = -2$? **Justify your answer properly, but briefly.** **SCORE: _____ / 4 PTS**

(1) $y' = (1 + x + y)^{\frac{1}{3}} = f$

(1) $f_y = \frac{1}{3}(1 + x + y)^{-\frac{2}{3}}$ IS NOT DEFINED @ $(1, -2)$ SINCE $(1 + 1 - 2) = 0$ AND $0^{-\frac{2}{3}}$ IS UNDEFINED

SO f_y IS NOT CONTINUOUS AROUND $(1, -2)$ (1)

SO E+U TELLS US NOTHING (1)

NO POINTS FOR THIS STEP IF YOU WROTE "E+U TELLS US THERE IS NO SOLUTION" OR THAT "THERE ARE MULTIPLE SOLUTIONS"